Eur. Phys. J. C 23, 113–120 (2002)
Digital Object Identifier (DOI) 10.1007/s100520100853

THE EUROPEAN PHYSICAL JOURNAL C

Extracting the CP-conserving and CP-violating form factors of $K_L \to \gamma\gamma$, $K_S \to \gamma\gamma$, and $\pi^0 \to \gamma\gamma$ from the decays of K_L, K_S and π^0 into two identical pairs of leptons

Z.E.S. Uy

Department of Physics, Millersville University, Millersville, PA 17551, USA

Received: 11 September 2001 /
Published online: 14 December 2001 – © Springer-Verlag / Società Italiana di Fisica 2001

Abstract. The extractions of the CP-conserving and CP-violating form factors of the decays of the long-lived kaon K_L and the short-lived kaon K_S into two photons from the branching ratios of $K_{S,L} \to \gamma\gamma$ and $K_{S,L} \to \gamma\gamma \to l^-l^+l^-l^+$, where l represents either an electron or a muon, are described. Using the currently available measurements of the branching ratios of $K_L \to \gamma\gamma$ and $K_L \to e^-e^+e^-e^+$ and assuming that the form factors are constants, the ratio of the CP-violating decay rate to the CP-conserving decay rate for $K_L \to \gamma\gamma$ is calculated to be 2.51% and the predicted branching ratio for $K_L \to \gamma\gamma \to \mu^-\mu^+\mu^-\mu^+$ is 6.07×10^{-13}. For $K_S \to \gamma\gamma$, a measurement of the branching ratio of $K_S \to \gamma\gamma \to e^-e^+e^-e^+$ is needed to determine the ratio of the CP-violating decay rate to the CP-conserving decay rate and to accurately predict the branching ratio of $K_S \to \gamma\gamma \to \mu^-\mu^+\mu^-\mu^+$. In the limit of complete CP-conservation the branching ratios are evaluated to be 4.39×10^{-10} for $K_S \to \gamma\gamma \to e^-e^+e^-e^+$ and 6.96×10^{-15} for $K_S \to \gamma\gamma \to \mu^-\mu^+\mu^-\mu^+$. In a similar model where the form factors of $\pi^0\gamma\gamma$ vertex are constants, the CP-violating decay rate component in $\pi^0 \to \gamma\gamma$ is shown, based on current available experimental data, to be less than 1% of the total decay rate.The magnitudes of the form factors are obtained by correlating the processes $\pi^0 \to \gamma\gamma$ and $\pi^0 \to \gamma\gamma \to e^-e^+e^-e^+$.

1 Introduction

This paper describes how the CP-conserving and CP-violating form factors in the decays of the long-lived kaon K_L and the short-lived kaon K_S into two photons can be extracted from the branching ratios of the decays of K_L and K_S into two identical lepton pairs $(l^-l^+\,;\,l^-l^+)$ via the double internal conversions in $K_{S,L} \to \gamma\gamma \to l^-l^+l^-l^+$. The electron pairs can either be electrons $(e^-e^+;$ $e^-e^+)$ or muons $(\mu^-\mu^+\,;\,\mu^-\mu^+)$.We will use K to represent either K_L or K_S in discussions that are germane to both of them.

This work also discusses the angular asymmetry function $\Delta(K \to \gamma\gamma \to l^-l^+l^-l^+)$ which reveals the degree of CP violation in $K \to \gamma\gamma$. The asymmetry function Δ depends on the angle ϕ which is the angle between the two planes of the lepton pairs, as shown in Fig. 1.

Although in an earlier paper [1], the author had discussed how the process $K \to \gamma\gamma \to e^-e^+\mu^-\mu^+$ can be utilized to achieve the same purpose, this paper has the advantage of using the existing measured values of the branching ratio (BR) of $K_L \to e^-e^+e^-e^+$ [2,3,4] to illustrate the extraction which enables the prediction of the branching ratio of $K_L \to \gamma\gamma \to \mu^-\mu^+\mu^-\mu^+$. In anticipation of the future measurements [5] of the branching ratios $BR(K_S \to e^-e^+e^-e^+)$ and $BR(K_S \to \mu^-\mu^+\mu^-\mu^+)$, the computations necessary to obtain the form factors of $K_S \to \gamma\gamma$ and the corresponding asymmetry functions are also presented.

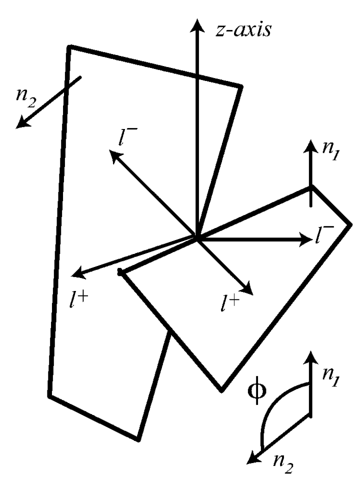

Fig. 1. The planes of the two pairs of leptons $(l^-l^+;\,l^-l^+)$ and the angle ϕ between the planes. The vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are the normals to the planes

The decay mode $\pi^0 \to \gamma\gamma$ has played an important role in establishing the existence of the triangular axial anomalies introduced by fermion loops which subsequently led to the conclusion that the quarks must come in three col-

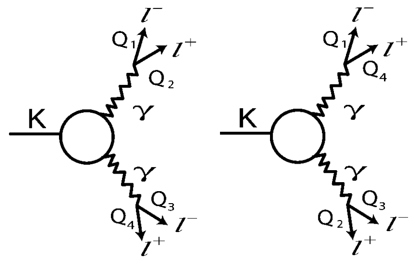

Fig. 2. Feynman diagrams for the process $K \to \gamma\gamma \to l^- l^+ l^- l^+$ where K is either the short-lived kaon K_S or the long-lived kaon K_L . The lepton l is either an electron or a muon

ors for the anomalies to cancel [6 - 9]. In this paper, we also deal with determining how much is the CP-violation content in the process $\pi^0 \to \gamma\gamma$ by correlating it with the double Dalitz mode $\pi^0 \to \gamma\gamma \to e^- e^+ e^- e^+$. The approach here is similar to the technique used by the author in the case of the neutral kaons K_L and K_S . The only published measurement of the rate of $\pi^0 \to e^- e^+ e^- e^+$ was performed by Samios et al. in 1962 [10]. There is a preliminary result announced by Fermilab's KTeV group in 1998 [11] and the analysis of the KTeV data taken during 1999 is in progress [12].

The discussion and analysis for the neutral kaons will be presented first followed by a similar exposition for the neutral pion.

In this paper, we emphasize that BR means branching ratio relative to the *total* decay rate.

2 Form factors of $K \to \gamma\gamma$ and asymmetry function of $K \to \gamma\gamma \to l^- l^+ l^- l^+$

One considers the Feynman graphs in Fig. 2 for the $K \to \gamma\gamma \to l^- l^+ l^- l^+$ decay process. Q_1, Q_2, Q_3, and Q_4 are the momenta of the leptons. M_K and m are the masses of the kaon and the lepton, respectively. The momenta of the two photons are k_1 and k_2. The QED coupling ie is assumed for the $\gamma l^- l^+$ vertex. For the $K\gamma\gamma$ vertex, it is assumed that the phenomenological Lagrangian [13,14]

$$L = \frac{iH}{4M_K} \Phi\, \varepsilon_{\mu\nu\alpha\beta}\, F_{\mu\nu} F_{\alpha\beta} + \frac{iG}{4M_K} \Phi F_{\mu\nu} F_{\mu\nu} \qquad (1)$$

holds. Φ is the meson field and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, where A_μ is the photon field. Both H and G are dimensionless form factors that parametrize the dynamics of the $K\gamma\gamma$ vertex. These form factors are, in general, dependent on the momenta of the two photons. Dynamical models where the $K\gamma\gamma$ vertex is momentum dependent have been proposed [15-22]. In particular, most of the analyses on $K_L\gamma\gamma$ vertex are geared toward elucidating the extent of the long-distance contribution of the 2γ intermediate state to the process $K_L \to \mu^- \mu^+$, a decay mode which probes second order processes in the Standard Model [23,24]. In this paper, it will be assumed that the momentum dependence can be neglected within the range of the energy that is involved:

$$\begin{aligned} H(k_1^2,\, k_2^2) &\approx H(0,\, 0) \\ G(k_1^2,\, k_2^2) &\approx G(0,\, 0) \end{aligned} \qquad (2)$$

for

$$\begin{aligned} 0 \le -k_1^2 &\le (M_K - 2m)^2 \\ 0 \le -k_2^2 &\le (M_K - 2m)^2 \;. \end{aligned} \qquad (3)$$

This assumption possibly does constrain the accuracy of the predictions of the model and the rendered computations. However, the magnitude of the potential deviations from the prospective results of more precise measurements of the pertinent decay rates that could ensue from the assumption can serve as a guide in constructing dynamical models where the exact momentum dependence of these form factors are delineated.

The invariant matrix element [25] is

$$\mathfrak{M} = \mathfrak{M}_1 + \mathfrak{M}_2 \qquad (4)$$

where

$$\begin{aligned} \mathfrak{M}_1 = {} & \frac{(-i\,e)^2}{(Q_1+Q_2)^2\,(Q_3+Q_4)^2} \left(\frac{-2\,i}{M_K}\right) \bar{u}(Q_1)\,\gamma_\alpha\, v(Q_2) \\ & \Big[H\, \delta_{\alpha\nu}\, \varepsilon_{\mu\nu\rho\sigma} (Q_1+Q_2)_\mu\, (Q_3+Q_4)_\rho\, \delta_{\sigma\beta} \\ & + G\,(Q_1+Q_2)\cdot(Q_3+Q_4)\,\delta_{\alpha\beta} \Big] \bar{u}(Q_3)\gamma_\beta v(Q_4) \end{aligned} \qquad (5)$$

and

$$\begin{aligned} \mathfrak{M}_2 = {} & \frac{(-i\,e)^2}{(Q_1+Q_4)^2\,(Q_3+Q_2)^2} \left(\frac{-2\,i}{M_K}\right) \bar{u}(Q_1)\,\gamma_\alpha\, v(Q_4) \\ & \Big[H\, \delta_{\alpha\nu}\, \varepsilon_{\mu\nu\rho\sigma} (Q_1+Q_4)_\mu\, (Q_3+Q_2)_\rho\, \delta_{\sigma\beta} \\ & + G\,(Q_1+Q_4)\cdot(Q_3+Q_2)\,\delta_{\alpha\beta} \Big] \bar{u}(Q_3)\gamma_\beta v(Q_2) \end{aligned} \qquad (6)$$

Since the process is a four-body decay, five independent variables are needed [26].The initially chosen variables are

$$\begin{aligned} x_2 &= -(Q_1+Q_2)^2 \\ x_3 &= -(Q_3+Q_4)^2 \\ y_2 &= -(Q_1+Q_2+Q_4)^2 \\ y_3 &= -(Q_1+Q_3+Q_4)^2 \\ w_{23} &= -(Q_2+Q_4)^2 \;\;. \end{aligned} \qquad (7)$$

To simplify the expression for $|\,\mathfrak{M}\,|^2$, the square of the absolute value of the invariant matrix element, we use the generic functions $\lambda(x,\, y,\, z,)$ and $\eta(x;\, y,\, z;\, u,\, v;\, w)$ where

$$\lambda(x,\, y,\, z) = x^2 + y^2 + z^2 - 2\,x\,y - 2\,x\,z - 2\,y\,z \qquad (8)$$

$$\begin{aligned} & \eta(x;\, y,\, z;\, u,\, v;\, w] \qquad (9) \\ & = -\Big[x^2 - x\,(y+z+u+v-w) + (y-z)\,(u-v) \Big] \,. \end{aligned}$$

The following terms are also employed in the expression for $|\,\mathfrak{M}\,|^2$:

$$\begin{aligned}
s &= (M_K)^2\\
z &= m^2\\
a_0 &= \eta(s;\, z,\, y_3;\, z,\, y_2;\, 8\, z)\\
a_2 &= \eta(s;\, z,\, y_2;\, x_3,\, x_2;\, 2\, z)\\
a_3 &= \eta(s;\, z,\, y_3;\, x_2,\, x_3;\, 2\, z)\\
b_2 &= \lambda(s,\, x_2,\, x_3)\, \lambda(y_2,\, z,\, s) - (a_2)^2\\
b_3 &= \lambda(s,\, x_3,\, x_2)\, \lambda(y_3,\, z,\, s) - (a_3)^2\\
a &= [a_2\, a_3 - \lambda(s,\, x_2,\, x_3)\, a_0] \,/\, [2\, s\, \lambda(s,\, x_2,\, x_3)]\\
b &= \sqrt{b_2\, b_3} \,/\, [2\, s\, \lambda(s,\, x_2,\, x_3)]\\
\sigma &= \lambda(s,\, x_2,\, x_3)\, b^2/16 \quad .
\end{aligned} \tag{10}$$

In the rest frame of the kaon, it can be shown [26] that w_{23} is linearly related to ϕ, the angle between the plane determined by Q_1 and Q_2 and the plane spanned by Q_3 and Q_4:

$$w_{23} = a + b\, \cos\phi \tag{11}$$

We can therefore use the variables $x_2,\, x_3,\, y_2,\, y_3,$ and ϕ as the five independent variables in the final expression for $\langle|\mathfrak{M}|^2\rangle$, the $|\mathfrak{M}|^2$ summed over all the spin states.

The differential decay width, expressed in terms of the variables $x_2,\, x_3,\, y_2,\, y_3,$ and ϕ, is

$$\begin{aligned}
d\Gamma &= \frac{1}{(128)\,(2\pi)^6\, s\sqrt{s}\, \sqrt{\lambda(s,\, x_2,\, x_3)}\, 2!\, 2!}\\
&\quad \times \left\langle|\,\mathfrak{M}\,|^2\right\rangle\, dx_2\, dx_3\, dy_2\, dy_3\, d\phi
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
\left\langle\,|\,\mathfrak{M}\,|^2\right\rangle &= 2\, \left(\frac{2\, e^2}{M_K}\right)^2\, \Big[|H|^2\, A(x_2,\, x_3,\, y_2,\, y_3,\, \phi)\\
&\quad +|G|^2\, B(x_2,\, x_3,\, y_2,\, y_3,\, \phi)\\
&\quad +\mathrm{Im}(H\, G^*)\, C(x_2,\, x_3,\, y_2,\, y_3,\, \phi)\Big]
\end{aligned} \tag{13}$$

and

$$\begin{aligned}
&A(x_2,\, x_3,\, y_2,\, y_3,\, \phi)\\
&= \Big\{2\, \lambda(s,\, x_2,\, x_3)\, \Big[2\, z\, (x_2 + x_3) - x_2\, x_3\Big]\\
&+4\, (16)\, \sigma\, \sin^2\phi\, - 8\, (x_2\, x_3)^2\\
&+4\, x_2\, x_3\, \Big[(y_2 + x_3 - s - z)\, (y_2 - x_2 - z)\\
&+(y_3 + x_2 - s - z)\, (y_3 - x_3 - z)\\
&+(x_2 + x_3 - s)^2\Big]\Big\}\, \Big[(x_2\, x_3)^2 \sqrt{\lambda(s,\, x_2, x_3)}\Big]^{-1}
\end{aligned} \tag{14}$$

$$\begin{aligned}
&B(x_2,\, x_3,\, y_2,\, y_3,\, \phi)\\
&= 2\, (x_2 + x_3 - s)^2\, \Big[2\, z\, (x_2 + x_3)\\
&+(y_2 + y_3 - s - a - b\, \cos\phi)\, (2\, z - a - b\, \cos\phi)\\
&+(x_2 - y_2 - z + a + b\, \cos\phi)\, (x_3 - y_3 - z + a + b\, \cos\phi)\Big]\\
&\times \Big[(x_2\, x_3)^2 \sqrt{\lambda(s,\, x_2, x_3)}\Big]^{-1}
\end{aligned} \tag{15}$$

$$\begin{aligned}
&C(x_2,\, x_3,\, y_2,\, y_3,\, \phi)\\
&= 8\, (x_2 + x_3 - s)\, \Big[x_2 + x_3 + s - 2\, y_2 - 2\, y_3 - 4\, z\\
&+4\, (a + b\, \cos\phi)\Big]\, (\sqrt{\sigma}\, \sin\phi)\, \Big[(x_2\, x_3)^2\, \sqrt{\lambda(s,\, x_2, x_3)}\Big]^{-1}\\
&+8\, \Big\{4\, z + \frac{1}{2}\, \Big[2\, z\, + y_2 + y_3 - s - 2\, (a + b\, \cos\phi)\Big]\Big\}\\
&\times \Big\{6\, z + y_2 + y_3 - 2\, x_2 - 2\, x_3 - 2\, (a + b\, \cos\phi)\Big\}\\
&\times (\sqrt{\sigma}\, \sin\phi)\, \Big[(x_2\, x_3)\, (3\, z + y_3 - x_3 - a - b\, \cos\phi)\\
&\cdot (3\, z + y_2 - x_2 - a - b\, \cos\phi)\, \sqrt{\lambda(s,\, x_2,\, x_3)}\Big]^{-1} \quad .
\end{aligned} \tag{16}$$

To obtain the angular spectrum distribution $\frac{d\Gamma}{d\phi}$, the variables $y_3,\ y_2,\ x_3,$ and x_2 are integrated out, in that order, with the following limits of integration [26]:

$$\begin{aligned}
y_2^{(\pm)} &= \Big\{\, \eta(x_3;\, z,\, z;\, s,\, x_2;\, 0)\\
&\qquad \pm\Big[\lambda(x_3,\, z,\, z)\, \lambda(x_3,\, s,\, x_2)\Big]^{\frac{1}{2}}\Big\} \Big/ (2\, x_3)\\
y_3^{(\pm)} &= \Big\{\, \eta(x_2;\, z,\, z;\, s,\, x_3;\, 0)\\
&\qquad \pm\Big[\lambda(x_2,\, z,\, z)\, \lambda(x_2,\, s,\, x_3)\Big]^{\frac{1}{2}}\Big\} \Big/ (2\, x_2)\\
x_3^{(-)} &= 4\, z\\
x_3^{(+)} &= (\sqrt{s} - \sqrt{x_2})^2\\
x_2^{(-)} &= 4\, z\\
x_2^{(+)} &= (\sqrt{s} - 2\, \sqrt{z})^2 \quad .
\end{aligned} \tag{17}$$

Henceforth, any integration with respect to any of the variables $x_2,\ x_3,\ y_2,$ and y_3 is understood to be subject to the above limits of integration.

Designating the integrals of $A,\ B,$ and C with respect to $x_2,\ x_3,\ y_2,$ and y_3 as $\alpha(\phi),\ \beta(\phi),$ and $\gamma(\phi)$ respectively,

$$\alpha(\phi) = \iiiint dx_2\, dx_3\, dy_2\, dy_3\, A(x_2,\, x_3,\, y_2,\, y_3,\, \phi) \tag{18}$$

$$\beta(\phi) = \iiiint dx_2\, dx_3\, dy_2\, dy_3\, B(x_2,\, x_3,\, y_2,\, y_3,\, \phi) \tag{19}$$

$$\gamma(\phi) = \iiiint dx_2\, dx_3\, dy_2\, dy_3\, C(x_2,\, x_3,\, y_2,\, y_3,\, \phi) \tag{20}$$

we can rewrite (12) as

$$\frac{d\Gamma}{d\phi} = F\, \Big[|H\,|^2\, \alpha(\phi) + |G\,|^2\, \beta(\phi) + \mathrm{Im}(H\, G^*)\, \gamma(\phi)\Big] \tag{21}$$

where

$$F = \left(\frac{2}{128}\right)\, \left(\frac{1}{2\,\pi}\right)^6\, \frac{e^4}{M_K^5} \quad . \tag{22}$$

The integrals of $\alpha(\phi)$ and $\beta(\phi)$ with respect to ϕ are needed to get the decay rate $\Gamma(K \to \gamma\gamma \to l^- l^+ l^- l^+)$. The integral of $\gamma(\phi)$ with respect to ϕ is zero. Note also that because $\gamma(\phi)$ is an odd function of ϕ,

$$\gamma(\phi) - \gamma(-\phi) = 2\, \gamma(\phi) \tag{23}$$

Letting

$$p = F \int_0^{2\pi} d\phi\, \alpha(\phi) \tag{24}$$

$$q = F \int_0^{2\pi} d\phi\, \beta(\phi) \tag{25}$$

and using these expressions in (21), we obtain

$$\Gamma(K \to \gamma\gamma \to l^- l^+ l^- l^+) = p\,|H\,|^2 + q\,|G\,|^2 \ . \tag{26}$$

From the Lagrangian in (1), the decay rate of $K \to \gamma\gamma$ is

$$\Gamma(K \to \gamma\gamma) = \frac{M_K}{16\,\pi}(|H\,|^2 + 2\,|G\,|^2) \ . \tag{27}$$

For K_L, H is the CP-conserving form factor and G is the CP-violating one; whereas for K_S, G is the CP-conserving form factor and H is the CP-violating one.

Defining the asymmetry function $\Delta(K \to \gamma\gamma \to l^- l^+ l^- l^+)$ as

$$\Delta(K \to \gamma\gamma \to l^- l^+ l^- l^+) = \left[\frac{d\Gamma}{d\phi}(\phi) - \frac{d\Gamma}{d\phi}(-\phi)\right] [\Gamma(K \to \gamma\gamma)]^{-1} , \tag{28}$$

we see that it gauges the asymmetry of the angular decay spectrum with respect to $\phi = 0$. Using (21) and (27) in the right hand side of (28), we obtain

$$\Delta(K \to \gamma\gamma \to l^- l^+ l^- l^+) = [2\,F\ \mathrm{Im}\,(H\,G^*)\ \ \gamma(\phi)] \left[\left(\frac{M_K}{16\,\pi}\right)(|H\,|^2 + 2\,|G\,|^2)\right]^{-1} . \tag{29}$$

We would like to express (26), (27), and (29) in terms of the moduli and relative phase difference of H and G. Letting

$$\begin{aligned} H &= h\ \exp[i\,\psi_h] \\ G &= g\ \exp[i\,\psi_g] \\ \delta &= (\psi_g - \psi_h)\,, \end{aligned} \tag{30}$$

(26) and (27) become

$$\Gamma(K \to \gamma\gamma \to l^- l^+ l^- l^+) = p\,h^2 + q\,g^2 \tag{31}$$

$$\Gamma(K \to \gamma\gamma) = \frac{M_K}{16\,\pi}(h^2 + 2\,g^2) \tag{32}$$

so that the branching ratio of $K \to \gamma\gamma \to l^- l^+ l^- l^+$ is

$$\begin{aligned} BR(K \to \gamma\gamma \to l^- l^+ l^- l^+) &= \frac{\Gamma(K \to \gamma\gamma \to l^- l^+ l^- l^+)}{\Gamma(K \to \gamma\gamma)} \cdot BR(K \to \gamma\gamma) \\ &= (p\,h^2 + q\,g^2)\left[\frac{M_K}{16\,\pi}(h^2 + 2\,g^2)\right]^{-1} BR(K \to \gamma\gamma)\,. \end{aligned} \tag{33}$$

From (31) and (32), we can solve for h^2 and g^2; this leads to the following results:

$$\begin{aligned} h^2 = &\left[q\,BR(K \to \gamma\gamma) - 2\left(\frac{M_K}{16\,\pi}\right) \times BR(K \to \gamma\gamma \to l^- l^+ l^- l^+)\right] \\ &\times [\Gamma(K \to all)]\left[\frac{M_K}{16\,\pi}(q - 2\,p)\right]^{-1} \end{aligned} \tag{34}$$

and

$$\begin{aligned} g^2 = -&\left[p\,BR(K \to \gamma\gamma) - \left(\frac{M_K}{16\,\pi}\right) \times BR(K \to \gamma\gamma l^- l^+ l^- l^+)\right] \\ &\times [\Gamma(K \to all)]\left[\frac{M_K}{16\,\pi}(q - 2\,p)\right]^{-1} . \end{aligned} \tag{35}$$

Meanwhile, the asymmetry function of (29) becomes

$$\Delta(K \to \gamma\gamma \to l^- l^+ l^- l^+) = -2\,F\,\left(\frac{16\,\pi}{M_K}\right)\left(\frac{h\,g\,\sin\delta}{h^2 + 2\,g^2}\right)\gamma(\phi)\ . \tag{36}$$

In (34) and (35), the values of p, q, and the coefficients of $\sin\phi$ and $\sin\phi\,\cos\phi$ in $\gamma(\phi)$ can be computed for both the electronic mode and the muonic mode. The author has used the software *Mathematica*[27] to carry out the symbolic and numerical computations for them; the results are in Table I.Using the tabulated values, we have for the electronic mode,

$$\begin{aligned} h^2 = &\Big[15.656 \times 10^{-2}\,BR(K \to \gamma\gamma) \\ &-8.800 \times 10^2 BR(K \to \gamma\gamma \to e^- e^+ e^- e^+)\Big] \\ &\times \Gamma(K \to all)\,(MeV)^{-1} \end{aligned} \tag{37}$$

$$\begin{aligned} g^2 = -&\Big[2.778 \times 10^{-2}\,BR(K \to \gamma\gamma) \\ &-4.440 \times 10^2 BR(K \to \gamma\gamma \to e^- e^+ e^- e^+)\Big] \\ &\times \Gamma(K \to all)\,(MeV)^{-1} \end{aligned} \tag{38}$$

$$\begin{aligned} \Delta(K \to \gamma\gamma \to e^- e^+ e^- e^+) &= \frac{h\,g\,\sin\delta}{(h^2 + 2\,g^2)}\,[1.639\sin\phi + 0.0026\,\sin\phi\,\cos\phi] \\ &\times 10^{-5}(rad)^{-1} \end{aligned} \tag{39}$$

and for the muonic mode,

$$\begin{aligned} h^2 = &\Big[15.656 \times 10^{-2} BR(K \to \gamma\gamma) \\ &-5.604 \times 10^7 BR(K \to \gamma\gamma \to \mu^- \mu^+ \mu^- \mu^+)\Big] \\ &\times \Gamma(K \to all)\,(MeV)^{-1} \end{aligned} \tag{40}$$

$$\begin{aligned} g^2 = -&\Big[2.778 \times 10^{-2} BR(K \to \gamma\gamma) \\ &-2.802 \times 10^7 BR(K \to \gamma\gamma \to \mu^- \mu^+ \mu^- \mu^+)\Big] \\ &\times \Gamma(K \to all)\,(MeV)^{-1} \end{aligned} \tag{41}$$

$$\Delta(K \to \gamma\gamma \to \mu^-\mu^+\mu^-\mu^+) = \frac{h\, g\, \sin\delta}{(h^2 + 2\, g^2)} [6.414\, \sin\phi + 0.2134\, \sin\phi\, \cos\phi] \times 10^{-12} (rad)^{-1} \quad . \tag{42}$$

3 Analysis for the neutral kaons

For K_L, the Particle Data Group [2] lists the branching ratios

$$BR(K_L \to \gamma\gamma) = (5.86 \pm 0.15) \times 10^{-4} \quad \text{(PDG)}$$
$$BR(K_L \to e^-e^+e^-e^+) = (4.1 \pm 0.8) \times 10^{-8} \ . \quad \text{(PDG)}$$

More recent values have been presented by Fermilab's KTeV/E799 group [3] and CERN's NA48 group [4]:

$$BR(K_L \to e^-e^+e^-e^+) = (3.72 \pm 0.18 \pm 0.23) \times 10^{-8} \quad \text{(KteV)}$$
$$BR(K_L \to e^-e^+e^-e^+) = (3.67 \pm 0.32 \pm 0.23 \pm 0.08) \times 10^{-8} \, . \quad \text{(NA48)}$$

Using the values $BR(K_L \to \gamma\gamma) = 5.86 \times 10^{-4}$ and $BR(K_L \to e^-e^+e^-e^+) = 3.83 \times 10^{-8}$, which is the average of the values given by PDG [2], KTeV [3] and NA48 [4], we obtain the following from (37) and (38):

$$h^2 = 73.501 \times 10^{-20} \tag{43}$$
$$g^2 = 0.9205 \times 10^{-20} \tag{44}$$

so that the ratio of the CP-violating rate to the CP-conserving rate in $K_L \to \gamma\gamma$ is

$$\frac{2\, g^2}{h^2} = 2.51\% \quad . \tag{45}$$

This amount of CP-violation is smaller than the CP-violating asymmetry of 13.6% detected by the KTeV's experiment on $K_L \to \pi^+\pi^-e^+e^-$ [28]. The asymmetry function is

$$\Delta(K_L \to \gamma\gamma \to e^-e^+e^-e^+) = \sin\delta_L\, [0.018\, \sin\phi + 1.774 \sin\phi\, \cos\phi] \times 10^{-6}\, (rad)^{-1} \, . \tag{46}$$

Using the values of h^2 and g^2 from (43) and (44), and the values of p and q for the muonic mode in Table I, one can predict from (31) the branching ratio of $K_L \to \gamma\gamma \to \mu^-\mu^+\mu^-\mu^+$:

$$BR(K_L \to \gamma\gamma \to \mu^-\mu^+\mu^-\mu^+) = 6.07 \times 10^{-13} \, . \tag{47}$$

Miyazaki and Takasugi [29] predicted the above branching ratio to be 5.54×10^{-13}, whereas Zhang and Goity [30], who employed chiral perturbation theory, predicted it to be 8×10^{-13}. The corresponding asymmetry function is

$$\Delta(K_L \to \gamma\gamma \to \mu^-\mu^+\mu^-\mu^+) = \sin\delta_L\, [6.30 \times 10^{-4}\, \sin\phi + 7.232\, \sin\phi\, \cos\phi] \times 10^{-13} (rad)^{-1} \, . \tag{48}$$

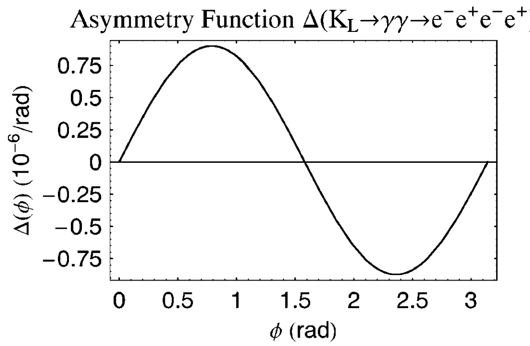

Fig. 3. The plot of $\Delta(K_L \to \gamma\gamma \to e^-e^+e^-e^+)$ for $\sin\delta_L = 1$. The maximum is at $\phi = 0.7890\, radian$, slightly to the right of $\pi/4 = 0.7854$; the minimum is at $\phi = 2.3598\, radian$, slightly to the right of $3\pi/4 = 2.3562$

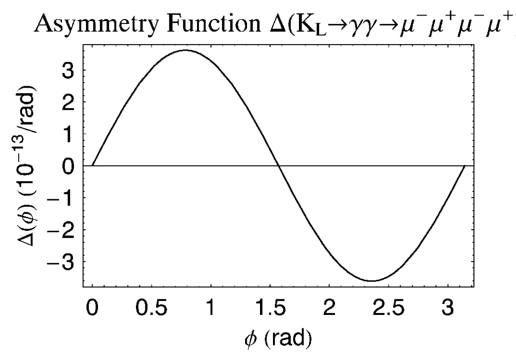

Fig. 4. The plot of $\Delta(K_L \to \gamma\gamma \to \mu^-\mu^+\mu^-\mu^+)$ for $\sin\delta_L = 1$.The maximum is at $\phi = 0.7854\, radian$; the minimum is at $\phi = 2.3562$

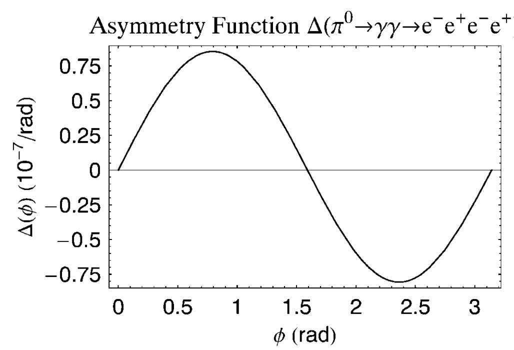

Fig. 5. The plot of $\Delta(\pi^0 \to \gamma\gamma \to e^-e^+e^-e^+)$ for $\sin\delta_\pi = 1$. The maximum is at $\phi = 0.793\, radian$, slightly to the right of $\pi/4 = 0.785\, radian$; the minimum is at $\phi = 2.363\, radian$, slightly to the right of $3\pi/4 = 2.356\, radian$

The plots of $\Delta(K_L \to \gamma\gamma \to e^-e^+e^-e^+)$ and $\Delta(K_L \to \gamma\gamma \to \mu^-\mu^+\mu^-\mu^+)$, with $\sin\delta_L = 1$, are shown in Fig. 3 and Fig. 4 respectively. The maximum of $\Delta(K_L \to \gamma\gamma \to e^-e^+e^-e^+)$ is $0.8998 \times 10^{-6}\, rad^{-1}$ at $\phi = 0.7890\, rad$, slightly skewed to the right of $\phi = \pi/4 = 0.7854\, rad$, while the minimum is $-\,0.8743 \times 10^{-6}\, rad^{-1}$ at $\phi = 2.3598\, rad$, slightly shifted to the right of $\phi = 3\pi/4 = 2.3562\, rad$. For $\Delta(K_L \to \gamma\gamma \to \mu^-\mu^+\mu^-\mu^+)$, the maximum is $3.6164 \times$

$10^{-13}(rad)^{-1}$ which occurs at $\phi = 0.7854\,rad$, while the minimum is -3.6156×10^{-13} at $\phi = 2.3562\,rad$.

In the case of the short-lived kaon K_S, the values of the branching ratio $BR(K_S \to \gamma\gamma)$ listed by the Particle Data Group [2] and reported recently by NA48 [31] are

$$BR(K_S \to \gamma\gamma) = (2.4 \pm 0.9) \times 10^{-6} \quad \text{(PDG)}$$
$$BR(K_S \to \gamma\gamma) = (2.58 \pm 0.36 \pm 0.22) \times 10^{-6} \quad \text{(NA48)}$$

yielding an average of $(2.49 \pm 0.74) \times 10^{-6}$. However, to date, there are no reported measurements of $BR(K_S \to e^-e^+e^-e^+)$ and $BR(K_S \to \mu^-\mu^+\mu^-\mu^+)$ so that we are unable to use (37) and (38) to extract the values of h^2 and g^2. The asymmetry function can be obtained from (39) and (42).

We can also establish the following ranges by imposing $h^2 \geq 0$ and $g^2 \geq 0$ in (37), (38), (40), and (41):

$$\begin{aligned} &0.626 \times 10^{-4}\, BR(K_S \to \gamma\gamma) \\ &\quad \leq BR(K_S \to \gamma\gamma \to e^-e^+e^-e^+) \\ &\quad \leq 1.76 \times 10^{-4}\, BR(K_S \to \gamma\gamma) \end{aligned} \tag{49}$$

$$\begin{aligned} &0.992 \times 10^{-9}\, BR(K_S \to \gamma\gamma) \\ &\quad \leq BR(K_S \to \gamma\gamma \to \mu^-\mu^+\mu^-\mu^+) \\ &\quad \leq 2.79 \times 10^{-9}\, BR(K_S \to \gamma\gamma) \end{aligned} \tag{50}$$

and using the average value of $BR(K_S \to \gamma\gamma) = 2.49 \times 10^{-6}$, we have

$$\begin{aligned} 1.56 \times 10^{-10} &\leq BR(K_S \to \gamma\gamma \to e^-e^+e^-e^+) \\ &\leq 4.39 \times 10^{-10} \end{aligned} \tag{51}$$

$$\begin{aligned} 2.47 \times 10^{-15} &\leq BR(K_S \to \gamma\gamma \to \mu^-\mu^+\mu^-\mu^+) \\ &\leq 6.96 \times 10^{-15}\,. \end{aligned} \tag{52}$$

In the limit when the CP-violating part is not present ($h = 0$), the average value of $BR(K_S \to \gamma\gamma)$, together with (37) and (40), yields $BR(K_S \to \gamma\gamma \to e^-e^+e^-e^+) = 4.39 \times 10^{-10}$ and $BR(K_S \to \gamma\gamma \to \mu^-\mu^+\mu^-\mu^+) = 6.96 \times 10^{-15}$.

4 Analysis for the neutral pion

The previous techniques can be applied to the process $\pi^0 \to \gamma\gamma \to e^-e^+e^-e^+$ provided the kaon's parameters are replaced with those of the pion's. Both H and G are dimensionless form factors that characterize the dynamics of the $\pi^0\gamma\gamma$ vertex; H is the CP-conserving form factor and G is the CP-violating one.We likewise assume the they are constants within the range of the energy involved. These form factors are, in general, dependent on the meomenta of the two photons. Chiral models where the $\pi^0\gamma\gamma$ vertex is discussed can be found in [6–9]. In particular, (34) and (35) translate into

$$\begin{aligned} h_\pi^2 = &\Big[q\, BR(\pi^0 \to \gamma\gamma) - 2\left(\frac{M_\pi}{16\,\pi}\right) \\ &\times BR(\pi^0 \to \gamma\gamma \to e^-e^+e^-e^+)\Big] \\ &\times [\Gamma(\pi^0 \to all)] \left[\frac{M_\pi}{16\,\pi}(q - 2\,p)\right]^{-1} \end{aligned} \tag{53}$$

and

$$\begin{aligned} g_\pi^2 = -&\Big[p\, BR(\pi^0 \to \gamma\gamma) - \left(\frac{M_\pi}{16\,\pi}\right) \\ &\times BR(\pi^0 \to \gamma\gamma \to e^-e^+e^-e^+)\Big] \\ &\times [\Gamma(\pi^0 \to all)] \left[\frac{M_\pi}{16\,\pi}(q - 2\,p)\right]^{-1} \end{aligned} \tag{54}$$

from which we deduce that

$$\begin{aligned} &p\left(\frac{16\pi}{M_\pi}\right) BR(\pi^0 \to \gamma\gamma) \\ &\quad \leq BR(\pi^0 \to \gamma\gamma \to e^-e^+e^-e^+) \\ &\quad \leq q\left(\frac{16\pi}{2\,M_\pi}\right) BR(\pi^0 \to \gamma\gamma) \end{aligned} \tag{55}$$

since $h_\pi^2 \geq 0$ and $g_\pi^2 \geq 0$.

The numerical computations for the values of p, q, and the coefficients of $\sin\phi$ and $\sin\phi\cos\phi$ in $\gamma(\phi)$ yield the results

$$p = 9.28 \times 10^{-5}\, MeV \tag{56}$$

$$q = 192.78 \times 10^{-5}\, MeV \tag{57}$$

$$\begin{aligned} \gamma(\phi) = &-(0.0482 \sin\phi + 2.346 \sin\phi\,\cos\phi) \\ &\times 10^{14}\, MeV^6\, rad^{-1}\,. \end{aligned} \tag{58}$$

The Particle Data Group [2] has listed the values $BR(\pi^0 \to \gamma\gamma) = (98.798 \pm 0.032) \times 10^{-2}$ and $\Gamma(\pi^0 \to all) = (7.8 \pm 0.6) \times 10^{-6}\, MeV$ while the KTeV group [11] has released the preliminary result $BR(\pi^0 \to e^-e^+e^-e^+) = (3.27 \pm 0.26) \times 10^{-5}$. Using these data and the above results in (55), we get

$$3.41 \times 10^{-5} \leq BR(\pi^0 \to \gamma\gamma \to e^-e^+e^-e^+) \leq 35.46 \times 10^{-5} \tag{59}$$

where the lower limit corresponds to $g_\pi^2 = 0$, the case when CP-violation is absent. This lower limit is within the range of the preliminary results from KTeV and is higher than the value 3.24×10^{-5} predicted by Miyazaki and Takasugi [29]. From (53) and (54), we obtain

$$h_\pi^2 = 2.87 \times 10^{-6} \tag{60}$$

$$g_\pi^2 = 0.52 \times 10^{-8} \tag{61}$$

$$\frac{2\,g_\pi^2}{h_\pi^2 + 2\,g_\pi^2} = 0.36 \times 10^{-2} = 0.36\%\,. \tag{62}$$

This says the CP-violating rate of $\pi^0 \to \gamma\gamma$ is less than 1% of its total decay rate. The asymmetry function is

$$\begin{aligned} &\Delta(\pi^0 \to \gamma\gamma \to e^-e^+e^-e^+) \\ &\quad = \sin\delta_\pi\,(0.034 \sin\phi + 1.66\,\sin\phi\,\cos\phi) \times 10^{-7}\, rad^{-1} \end{aligned} \tag{63}$$

which has maximum at $\phi = 0.793\,rad$, at the right of $\pi/4 = 0.785\,rad$, and minimum at $\phi = 2.363\; rad$, at the right of $3\pi/4 = 2.356\,rad$. Figure 5 shows the plot of Δ against ϕ for $\sin\delta_\pi = 1$.

The same technique can be applied to correlate $\eta \to \gamma\gamma$ with $\eta \to \gamma\gamma \to e^- e^+ e^- e^+$ and $\eta \to \gamma\gamma \to \mu^- \mu^+ \mu^- \mu^+$; the results in doing so are

$$2.55 \times 10^{-5} \le BR(\eta \to \gamma\gamma \to e^- e^+ e^- e^+) \le 6.60 \times 10^{-5} \quad (64)$$

$$2.54 \times 10^{-9} \le BR(\eta \to \gamma\gamma \to \mu^- \mu^+ \mu^- \mu^+) \le 6.56 \times 10^{-9} \quad (65)$$

where the lower limits refer to the case of no CP-violation or $g_\eta^2 = 0$. These values are higher than the corresponding values of 2.42×10^{-5} and 2.45×10^{-9} calculated by Miyazaki and Takasugi [29] under the assumption of no CP-violation.

The asymmetry functions are

$$\Delta(\eta \to \gamma\gamma \to e^- e^+ e^- e^+) = \frac{h_\eta \, g_\eta}{(h_\eta^2 + 2\, g_\eta^2)} \sin\delta_\eta \,(0.009 \sin\phi + 1.69 \sin\phi \cos\phi) \times 10^{-5}\, rad^{-1} \quad (66)$$

$$\Delta(\eta \to \gamma\gamma \to \mu^- \mu^+ \mu^- \mu^+) = \frac{h_\eta \, g_\eta}{(h_\eta^2 + 2\, g_\eta^2)} \sin\delta_\eta \,(0.115 \sin\phi + 8.32 \sin\phi \cos\phi) \times 10^{-11}\, rad^{-1} \quad . \quad (67)$$

In the absence of measurements of the decay rates of either $\eta \to e^- e^+ e^- e^+$ or $\eta \to \mu^- \mu^+ \mu^- \mu^+$, we are unable to extract the values of h_η^2 and g_η^2. Recently, Akhmetshin *et al.*[32] has established an upper limit of $BR(\eta \to e^- e^+ e^- e^+) < 6.9 \times 10^{-5}$ which is higher than the upper limit in (64).

5 Conclusion

We have presented a modus with which the CP-conserving and CP-violating form factors of $K_L \to \gamma\gamma$ and $K_S \to \gamma\gamma$ can be garnered from the branching ratios $BR(K_{S,L} \to \gamma\gamma)$ and $BR(K_{S,L} \to \gamma\gamma \to l^- l^+ l^- l^+)$ where l represents either an electron or a muon. The manifestation of CP-violation can also be detected by measuring the asymmetry function Δ in the decays $K_{S,L} \to \gamma\gamma \to l^- l^+ l^- l^+$. From the known values of $BR(K_L \to \gamma\gamma)$ and $BR(K_L \to e^- e^+ e^- e^+)$, we deduced that the ratio of the CP-violating part to the CP-conserving part in $K_L \to \gamma\gamma$ decay is 2.51% and the branching ratio $BR(K_L \to \gamma\gamma \to \mu^- \mu^+ \mu^- \mu^+)$ is 6.07×10^{-13}.

We await the future measurements [5] of the branching ratio and asymmetry function Δ of $K_S \to e^- e^+ e^- e^+$ to detect and gauge the degree of CP violation in $K_S \to \gamma\gamma$.

In a similar model where the form factors of $\pi^0\gamma\gamma$ vertex are constants, the CP-violating decay rate in $\pi^0 \to \gamma\gamma$ is shown, based on current available experimental data, to be less than 1% of the total decay rate. A way of detecting the presence of of CP violation in $\pi^0 \to \gamma\gamma$ and $\eta \to \gamma\gamma$ is to reconnoiter the double Dalitz modes $\pi \to \gamma\gamma \to e^- e^+ e^- e^+$ and $\eta \to \gamma\gamma \to e^- e^+ e^- e^+$ or $\eta \to \gamma\gamma \to \mu^- \mu^+ \mu^- \mu^+$ and measure their asymmetry functions.

Acknowledgements. The author gratefully acknowledges the hospitalities of the University of Pennsylvania and Fermilab. This work has been supported by the Faculty Grants Committee of Millersville University.

References

1. Z. E. S. Uy, Phys. Rev. D **43**, 802 (1991)
2. Particle Data Group, D. E. Groom et al., Eur. Phys. J. C **15**, 1 (2000)
3. KTeV Collaboration, A. Alavi-Harati et al., Phys. Rev. Lett. **86**, 5425 (2001); E. Halkiadakis, Ph.D. thesis, Rutgers University, 2001; P. Gu et al., Phys. Rev. Lett. **72**, 3000 (1994)
4. NA48 Collaboration, A. Bevan et al., J. Phys. G **27**, 1161 (2001)
5. P. S. Cooper, in Kaon Physics, edited by J. L. Rosner, B. D. Winstein (University of Chicago Press, Chicago, 2001), pp.571-579; G. Kalmus, ibid., pp.581-583
6. S. L.Adler, Phys. Rev. **177**, 2426 (1969); J. S. Bell, R. Jackiw, Nuovo Cim. **60A**, 47 (1969)
7. G. t'Hooft, in Proceedings of the NATO Advanced Study Institute, Cargese, 1979, edited by G. t'Hooft (Plenum, New York, 1980)
8. R. J. Jackiw, in Proceedings of the Symposium on Anomalies, Geometry and Topology, edited by W. A. Bardeen, A. R. White (World Scientific, Singapore, 1985)
9. R. E. Marshak, Conceptual Foundations of Modern Particle Physics (World Scientific, Singapore, 1993), pp.413-460
10. N. P. Samios, R. Plano, A. Prodell, M. Schwartz, J. Steinberger, Phys. Rev. **126**, 1844 (1962)
11. A. Baker, in Proceedings of the 29th International Conference on High Energy Physics, Vancouver, 1998, edited by A. Astbury, D. Axen, J. Robinson (World Scientific, Singapore, 1999)
12. A. Baker (email communication)
13. R. H. Dalitz, Proc. Phys. Soc. London A **64**, 667 (1951)
14. T. Miyazaki, Nuovo Cimento Lett. **5**, 125 (1972); **25**, 1 (1979)
15. L. Bergstrom, E. Masso, P. Singer, Phys. Lett. **131B**, 229 (1983)
16. L. Bergstrom, E. Masso, P. Singer, D. Wyler, Phys. Lett. **134B**, 373 (1984)
17. Z. E. S. Uy, Phys. Rev. D **3**, 234 (1971); **19**, 1623 (1979); **27**, 300 (1983); **29**, 574 (1984); **37**, 2684 (1988)
18. V. Barger, Nuovo Cimento **32**, 128 (1964); H. Stern, ibid., **51A**, 197 (1967); B. R. Martin, E. de Rafael, Nucl. Phys. **B8**, 131 (1968); K. Nishijima, Fields and Particles: Field Theory and Dispersion Relation (Benjamin, New York, 1969); L. M. Sehgal, Phys. Rev. **183**, 1511, (1969); B. R. Martin, E. de Rafael, J. Smith, Phys. Rev. D **2**, 179 (1970); J. Yndurain, Prog. Theor. Phys. **46**, 990 (1971); Y. Kohara, ibid., **48**, 261 (1972).
19. L. L. Chau, H. Y. Cheng, Phys.Rev. Lett. **54**, 1768 (1985); R. Decker, P. Pavlopoulos, G. Zoupanos, Z. Phys. C **28**, 117 (1985); A. N. Ivanov, N. I. Troitskaya, M. K. Volkov, Phys. Lett. B **175**, 467 (1986); G. D'Ambrosio, D. Espriu, ibid., **175**, 237 (1986); J. L. Goity, Z. Phys. C **34**, 341 (1987); L. L. Chau, H. Y. Cheng, Phys. Lett. B **195**, 275 (1987); J. Abad, R. Rodriguez-Trias, Z. Phys. C **41**, 341 (1988); I. Picek, Nucl. Phys. B (Proc. Suppl.) **24A**, 101 (1991)

20. E. Ma, A. Pramudita, Phys. Rev. D **24**, 2476 (1981); Y. Dupont, T. N. Pham, ibid., **29**, 1368 (1984)
21. J. L. Goity, L. Zhang, Phys. Lett. B **398**, 387 (1997)
22. G. D'Ambrosio, G. Isidori, J. Portoles, Phys. Lett. B **423**, 385 (1998)
23. L. M. Sehgal, Phys. Rev. **183**, 1511 (1969); M. K. Gaillard, B. W. Lee, Phys. Rev. D **9**, 897 (1974); M. K. Gaillard, B. W. Lee, R. E. Shrock, ibid., **13**, 2674 (1976); R. E. Shrock, M. B. Voloshin. Phys. Lett. **87B**, 375 (1979); T. Inami, C. S. Lim, Prog. Theor. Phys. **65**, 297 (1981)
24. C. Q. Geng, John N. Ng, Phys. Rev. D **41**, 2351 (1990); L. Bergstrom, E. Masso, P. Singer, Phys. Lett. B **249**, 141 (1990); G. Belanger, C. Q. Geng, Phys. Rev. D **43**, 140 (1991); G. Buchalla, A. J. Buras, M. K. Harlander, Nucl. Phys. B **349**, 1 (1991); P. Ko, Phys. Rev. D **45**, 174 (1992)
25. B. DeWit, J. Smith, Field Theory in Particle Physics,1st ed. (North-Holland, Amsterdam, 1986)
26. P. Nyborg, H. S. Soong, W. Kerman, R. H. Good, Jr., Phys. Rev. **140**, B914 (1965)
27. S. Wolfram, The Mathematica Book, 4th ed. (Cambridge University Press, Cambridge, 1999); Wolfram Reasearch, Mathematica 4.0, (Wolfram Media, Champaign, IL., 1999)
28. KTev Collaboration, A. Alavi-Harati et al., Phys. Rev. Lett. **84**, 408 (2000)
29. T. Miyazaki, E. Takasugi, Phys. Rev. D **8**, 2051 (1973)
30. L. Zhang, J. L. Goity, Phys. Rev. D **57**, 7031 (1998)
31. NA48 Collaboration, A. Bevan et al., Phys. Lett. B **493**, 29 (2000)
32. R. R. Akhmetshin et al., Phys. Lett. B **501**, 191 (2001)